[2] Francis, D. G., "Development of a Vortex Laser Doppler Velocimeter," *Proceedings of the 1971 National Aerospace Electronics Conference*, IEEE, AIAA, *1971*, Dayton, Ohio, pp. 36–40.

[3] Huffaker, R. M., Jelalian, A., Keene, W., Sonnenschein, C. and Thomson, J. A. L., "Applications of Laser Droppler Systems to Vortex Measurement and Detection," *Aircraft Wake Turbulence and its Detection*, 1st ed., Plenum Press, New York, 1971, pp. 113–124.

[4] Easterbrook, C. C. and Joos, N. W., "The Utility of Doppler Radar in the Study of Aircraft Wingtip Vortices," *Aircraft Wake Turbulence and its Detection*, 1st ed., Plenum Press, New York, 1971, pp. 97–112.

[5] Kodis, R. D., "Wake Vortex Sensors," *Proceedings of the FAA Symposium on Turbulence*," Federal Aviation Administration, March, 1971, Washington, D.C., p. 28.

Compressibility Correction for Internal Flow Solutions

S. Lieblein* and N. O. Stockman†
NASA Lewis Research Center Cleveland, Ohio

THERE are many instances in internal aerodynamics where high subsonic or local transonic flows are present. Examples of such configurations include engine nacelle inlets, lift fan and lift engine inlets, compressor and turbine blade rows, diffusers, transition ducts, and thrust deflection devices. Fairly accurate solutions of the flow distributions along the surfaces or across the flow passage are frequently desired for design and analysis purposes.

Unfortunately, accurate general compressible solutions are not readily available for these situations. However, incompressible potential flow solutions are generally tractable for many of these configurations. It would be helpful, therefore, if some simple general compressibility correction could be developed for use with these methods that would produce a relatively good approximation to the compressible flow behavior.

Compressibility corrections of various forms based on upstream freestream Mach number are in frequent use for flow around submerged bodies such as airfoil sections. Recent developments[1,2] have indicated the importance of including geometry factors in the compressibility correction relations. It is expected that similar geometry factors exist for compressibility effects in channels. Thus, the derivation of an effective compressibility correction for a wide range of internal flow configurations comparable to that for a submerged body[1,2] is likely to be a complex matter. It appeared expedient therefore to explore the possibility of devising a simple correction relation based on empirical observation.

This Note proposes a simple compressibility correction for internal flow solutions that may have promise as a preliminary approach to enlarging the usefulness of incompressible potential flow calculations. The correction equation was deduced from inspection of the exact solution for the compressible flow in a turbine nozzle passage as described in Ref. 3. The blade profile is shown in Fig. 1, and the exact compressible and incompressible surface velocities, as obtained from the method of Ref. 3, are plotted in Fig. 2. It is seen that the compressibility effect is not the same in magnitude for the suction and pressure surfaces.

A correction function for the local compressible velocity,

Received October 4, 1971.

* Chief, VTOL Propulsion Branch. Associate Fellow AIAA.
† Aerospace Research Engineer. Member AIAA.

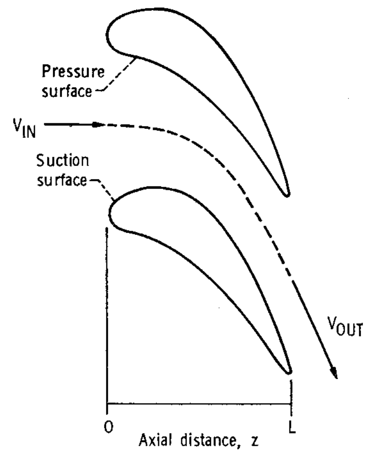

Fig. 1 Accelerating stator blade passage.

V_c, at any point as a function of the incompressible velocity, V_i, at the point was established as

$$V_c = V_i(\rho_i/\bar{\rho}_c)^{V_i/\bar{V}_i} \tag{1}$$

where: $\bar{V}_i$ = the average incompressible velocity across the flow passage at the given station; ρ_i = incompressible density, which is equal to the stagnation density, ρ_t; $\bar{\rho}_c$ = average compressible density across the flow passage.

In Eq. (1), the density ratio term represents the effect of average Mach number, while the passage surface geometry is reflected in the magnitude of the exponent $V_i/\bar{V}_i$. For the suction surface of the passage in Fig. 1, the exponent is greater than unity, while for the pressure surface, the value is less than unity.

The compressibility correction was applied to the geometry of Fig. 1 based on the incompressible values of surface velocity shown in Fig. 2. Average incompressible velocity, $\bar{V}_i$, was obtained from averaging the local values of V_i across the passage at each Z/L position. Figure 3 shows the comparison between the exact compressible velocities and the values given by Eq. (1) for the case where the average compressible density, $\bar{\rho}_c$, was also obtained from averaging the local values of exact ρ_c across the passage at each Z/L position. This actual average compressible density (which would not be available in a practical case) was used to assess the basic accuracy of the relation. The agreement is seen to be excellent

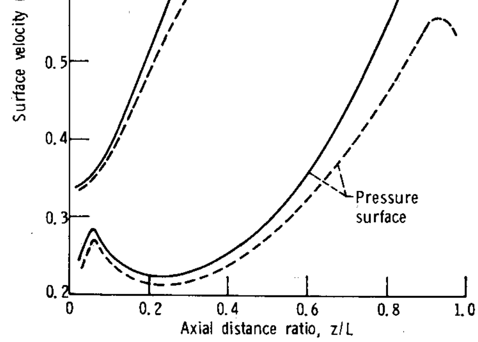

Fig. 2 Exact compressible and incompressible surface velocities for blade passage of Fig. 1 (method of Ref. 3).

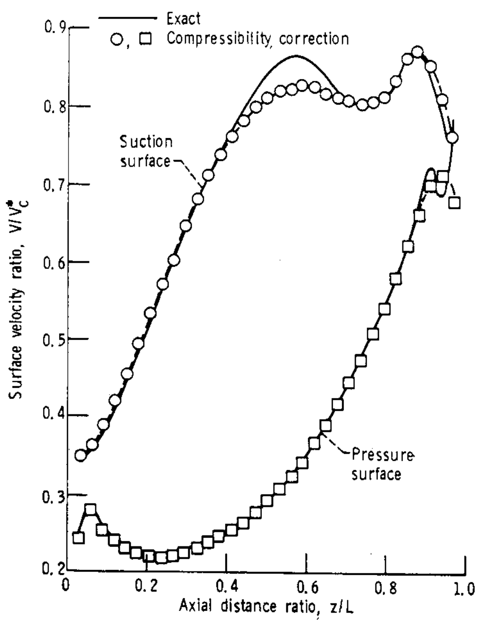

Fig. 3 Comparison of exact and compressibility correction values of surface velocity. Average compressible density from exact compressible density profile.

except for the region of the first velocity peak on the suction surface.

For the general case for which the exact compressible values are unknown, the average compressible density can be determined from the average incompressible velocity as follows. From continuity

$$\bar{V}_i/V_c^* = (\bar{\rho}_c/\rho_t)(\bar{V}_c/V_c^*) \tag{2}$$

where V_c^* is the critical velocity. For isentropic flow, the compressible critical velocity ratio can be expressed in terms of the density ratio

$$\bar{V}_c/V_c^* = \{[(\gamma + 1)/(\gamma - 1)][1 - (\bar{\rho}_c/\rho_t)^{\gamma-1}]\}^{1/2} \tag{3}$$

which, when combined with Eq. (2), yields

$$\bar{V}_i/V_c^* = (\bar{\rho}_c/\rho_t)\{[(\gamma + 1)/(\gamma - 1)][1 - (\bar{\rho}_c/\rho_t)^{\gamma-1}]\}^{1/2} \tag{4}$$

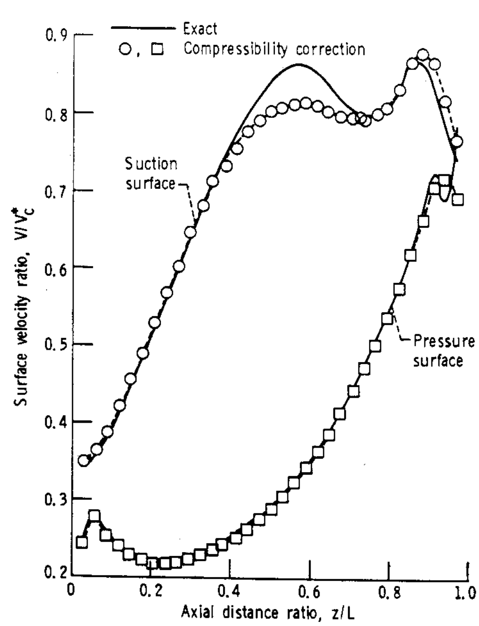

Fig. 4 Comparison of exact and compressibility correction values of surface velocity. Average compressible density from average incompressible velocity [Eq. (4)].

which now relates the average compressible density to the average incompressible velocity.

The comparison between the exact compressible velocities and the correction values with $\bar{\rho}_c$ obtained from Eq. (4) is shown in Fig. 4. Again, the agreement is considered excellent, except for the region of the first velocity peak on the suction surface.

The encouraging results obtained in the comparisons of Figs. 3 and 4 strongly suggest further investigation of the applicability of the devised compressibility correction for internal flow solutions.

References

[1] Norstrud, H., "A Correction for Compressible Subsonic Planar Flow," *Journal of Aircraft*, Vol. 8, No. 8, Feb. 1971, pp. 123–126.

[2] Willy, P. G., "The Calculation of Sub-Critical Pressure Distributions on Symmetric Aerofoils at Zero Incidence," NPL Aero Rept. 1208, March 1967, National Physics Lab., Teddington, England.

[3] Katsanis, T., "Fortran Program for Calculating Transonic Velocities on a Blade-to-Blade Stream Surface of a Turbomachine," TN D-5427, Sept. 1969, NASA.

Internal Parachute Flows

Paul C. Klimas*
U. S. Naval Academy, Annapolis, Md.

1. Introduction

THERE has been little success in theoretically predicting details of flowfields surrounding porous parachute canopies. The basic design objective of taking full advantage of fluid viscosity and use of a permeable material requires most analyses to be in terms of average quantities and heavily dependent upon empirical results. As a first step away from this approach, it can be asked if a potential flow model might reasonably predict some portion of the flowfield. One such method is described here with application to the steady incompressible flow internal to a porous canopy of arbitrary axisymmetric cross section.

2. Model

The particular model chosen was that of a vortex sheet placed to coincide with the physical location of the canopy. In cylindrical coordinates the Stoke's stream function corresponding to an axisymmetric vortex sheet placed in a uniform stream of velocity $-U$ is[1]

$$\psi = (-r/4\pi)\int_0^{2\pi}\cos\theta d\theta\int_0^{S_{max}}\gamma r_i[r^2 + r_i^2 + (z - z_i)^2 - 2rr_i\cos\theta]^{-\frac{1}{2}}ds_i + Ur^2/2$$

The i subscript refers to a sheet location,

$$ds_i = (dr_i^2 + dz_i^2)^{\frac{1}{2}}$$

and $\gamma = \gamma(s_i)$ is the unknown local sheet strength. Determination of this quantity is made algebraically. The canopy

Received October 21, 1971. Portions of this work were completed at the U. S. Army Airdrop Engineering Laboratory, Natick, Mass.

*Assistant Professor, Aerospace Engineering Department. Associate Member AIAA.